The New Mathematics
for Primary Teachers

2. Operations with Sets and Numbers

D. Paling and J. L. Fox

OXFORD UNIVERSITY PRESS

2465/21.3.75

THE NEW MATHEMATICS
FOR PRIMARY TEACHERS

2. Operations with Sets and Numbers

D. Paling and J. L. Fox

S.C.E.R.T., West
Library
Calcutta
25/3, B. C.

LONDON
OXFORD UNIVERSITY PRESS

OXFORD UNIVERSITY PRESS, ELY HOUSE, LONDON W. 1
GLASGOW NEW YORK TORONTO MELBOURNE WELLINGTON
CAPE TOWN IBADAN NAIROBI DAR ES SALAAM LUSAKA ADDIS ABABA
DELHI BOMBAY CALCUTTA MADRAS KARACHI LAHORE DACCA
KUALA LUMPUR SINGAPORE HONG KONG TOKYO

© Oxford University Press 1968

First published 1968
Reprinted 1971, 1972, 1973

S.C.E.R.T., West Bengal
Date 21.3.75
Acc. No. 2465

Designed by
DOUGLAS HALL

517.521
PAL

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior permission of Oxford University Press

Filmset by Oliver Burridge Filmsetting Ltd., Crawley, Sussex
Printed in Great Britain at the University Press, Oxford
by Vivian Ridler, Printer to the University

Preface to the series

At the present time there is a growing dissatisfaction with mathematical work in schools. There is a feeling that the content of courses does not fit in with the present-day needs of science and technology, industry and trade. At the same time the attitude of teachers towards the teaching of mathematics is undergoing a fundamental change. More and more teachers are coming to feel that teaching by 'rule' is not good enough. They want children to find out more for themselves, to question what they are doing and to find their own ways of solving problems. They want children to understand rather than to learn like parrots.

These two movements—the modernisation of the content of mathematics and the shift of emphasis from 'the teacher teaching' to 'the child learning'—are going on together in many countries. They both make large demands on teachers, especially in the primary school. These teachers now need to take a new look at mathematics itself and at the same time try to introduce more enlightened methods in the classroom.

If these changes are to come about, teachers in schools and students in training will need all the help they can be given. It is for this purpose that this series of short books has been written. The aim is to give teachers and student-teachers a new mathematical background so that they can introduce modern mathematics in the primary school. The books do not set out to deal with the classroom situation itself but they do present, at teacher level, the kind of approach now being introduced into schools.

The first five books will provide a background to the first three years of mathematical work in the primary school. Further books will cater for the whole primary range. The approach is simple and straightforward and no previous knowledge is assumed. Many exercises (with answers) are provided at each stage; they need to be worked through very carefully. The exercises not only consolidate the work already done but provide the starting-point for the next stage.

St Luke's College, Exeter
March, 1968

D.P.
J.L.F.

CONTENTS

1. UNION OF SETS 1
A. Sub-sets 1
B. Intersection of sets 2
C. Union of sets 6

2. THE UNION OF DISJOINT SETS 10
A. Disjoint sets 10
B. Union of more than two disjoint sets 12

3. ADDITION AND SUBTRACTION 14
A. Looking again at the union of disjoint sets; the idea of addition 14
B. The idea of subtraction 17
C. Using addition and subtraction 20

4. ADDITION, MULTIPLICATION, AND DIVISION 21
A. The addition of more than two numbers 21
B. The addition of equal numbers; the idea of multiplication 22
C. The idea of division 25
D. Using multiplication and division 26

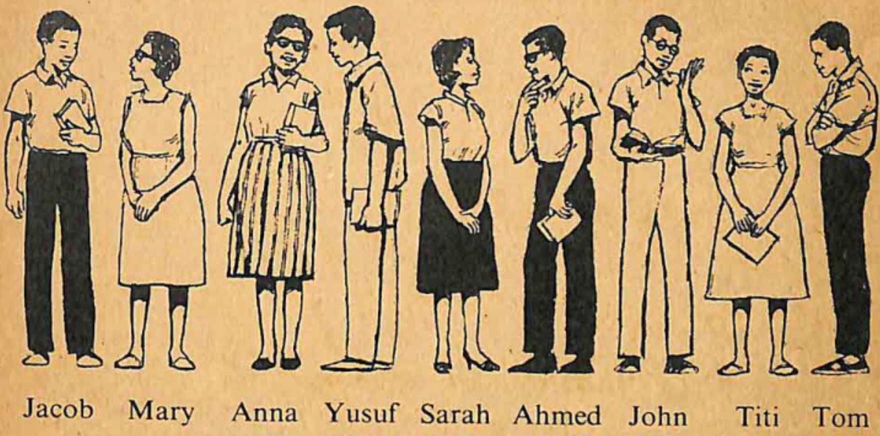
5. MORE ABOUT SETS AND NUMBERS 28
A. Operations 28
B. The commutative property 28
C. The associative property 30
D. The distributive property 32
E. Nought and one 33

ANSWERS 36

1. UNION OF SETS

A. Sub-sets

Here is a picture of some college students.



We notice that some of the students are men, some are women, some are carrying books, some are wearing spectacles. We remember, from Book 1, *Numbers and Number Systems*, that we can show the *set* of the names of the men students as:

{Jacob, Yusuf, Ahmed, John, Tom}

Exercises

1. Write the set of names of:
   (a) the woman students,
   (b) the students carrying books,
   (c) the students wearing spectacles,
   (d) the students not carrying books,
   (e) the students not wearing spectacles.
2. Write the set of the months of the year.
   Now write the set of months:
   (a) starting with 'J',
   (b) finishing with 'ber',
   (c) with 31 days,
   (d) with 30 days,
   (e) with 20 days.

In exercise 2 we started with the complete set of the months of the year. We then wrote down sets made up of some of the months. Each of these is called a sub-set of:

{January, February, March, April, May, June, July, August, September, October, November, December}

Sometimes a sub-set contains no members, as in exercise 2(e).

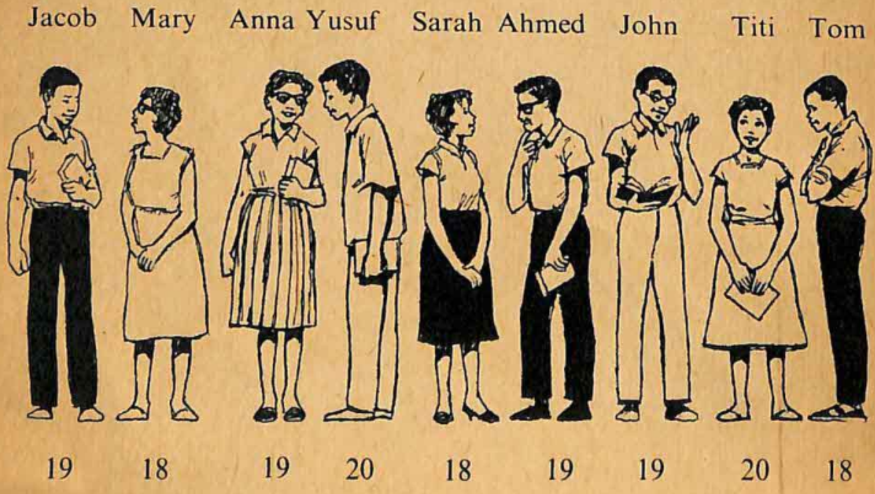
Exercises

3. For {1, 2, 3, 4, 5, 6, 7, 8, 9}, write the sub-set of:
   - (a) numbers greater than five,
   - (b) numbers less than five,
   - (c) even numbers,
   - (d) odd numbers,
   - (e) numbers greater than three and less than eight.

4. For {e, d, u, c, a, t, i, o, n}, write the sub-set of:
   - (a) vowels,
   - (b) consonants.

B. Intersection of sets

Here are the students again with their ages.



We can again show them in various sub-sets. For example:

Students aged 18 {Mary, Sarah, Tom}
Students aged 19 {Jacob, Anna, Ahmed, John}
Students aged 20 {Yusuf, Titi}
Men students {Jacob, Yusuf, Ahmed, John, Tom}
Women students {Mary, Anna, Sarah, Titi}
Women students aged 18 {Mary, Sarah}

There are several ways of finding the last of these sub-sets. We can:

(a) Look at the diagram, and pick out the women students {Mary, Anna, Sarah, Titi}.
Now see which are 18 {Mary, Sarah}.

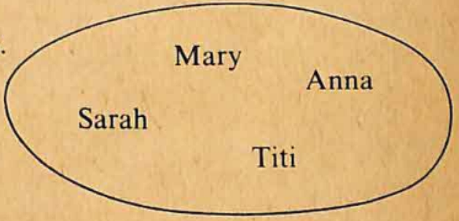
(b) Look at the diagram, and pick out the students who are 18 {Mary, Sarah, Tom}.
Now see which are women {Mary, Sarah}.

(c) Look at the set of students aged 18 {Mary, Sarah, Tom} and the set of women students {Mary, Anna, Sarah, Titi} and see which students appear in both sets {Mary, Sarah}.

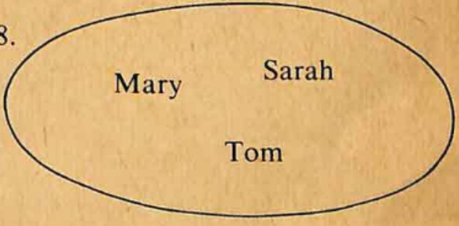
In each case we get {Mary, Sarah} as the set of women students who are 18.

Sometimes it is helpful to show this information in a diagram like this.

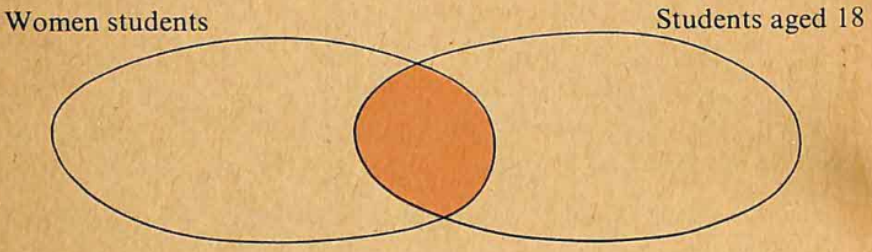
First we show the women students.



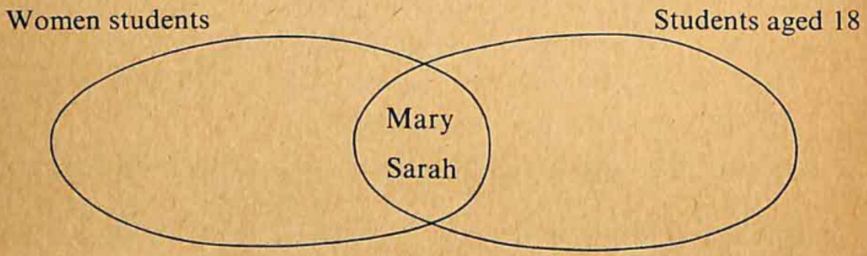
Then we show the students aged 18.



Now we want some way of showing that Mary and Sarah are in both sets. We do this by drawing the two shapes in this way:

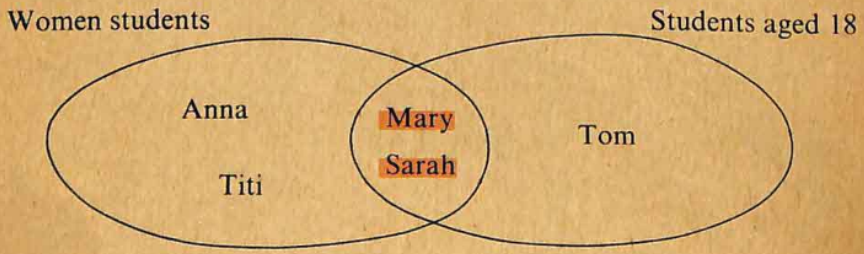


The shaded portion is in both shapes; it is where the shapes intersect. In it we put the students Mary and Sarah.



The diagram now shows that Mary and Sarah are women students and are also aged 18.

The other students are then put in and we have:



This kind of diagram is often called a Venn diagram. It helps us to see the information more clearly. Looking at it we can say:

(a) The women students are {Anna, Titi, Mary, Sarah}.
(b) The students, aged 18, are {Mary, Sarah, Tom}.
(c) Mary and Sarah are women and are aged 18.
(d) Tom is aged 18 and is a man.
(e) Anna and Titi are women and are not aged 18.

Let us now look back at what we have been doing. We started with {Anna, Titi, Mary, Sarah} and {Mary, Sarah, Tom}. We then saw that Mary and Sarah are in both sets. Finally we showed this in a diagram.

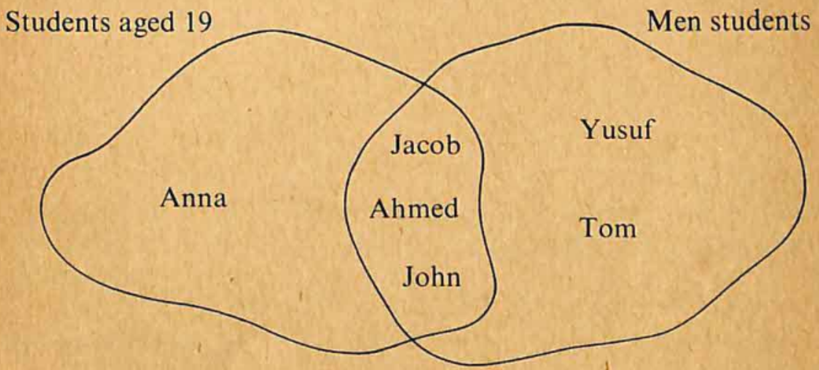
We use a special word to describe {Mary, Sarah}. We say {Mary, Sarah} is the intersection of {Anna, Titi, Mary, Sarah} and {Mary, Sarah, Tom}.

Now we look at:

Students aged 19 {Jacob, Anna, Ahmed, John}

Men students {Jacob, Yusuf, Ahmed, John, Tom}

We notice that Jacob, Ahmed, and John are in both sets. So that Jacob, Ahmed, and John are men students, aged 19. We can show this as:



We say 'The intersection of {Jacob, Anna, Ahmed, John} and {Jacob Yusuf, Ahmed, John, Tom} is {Jacob, Ahmed, John}.' We write this more shortly, using the symbol ∩ for 'intersection' and = for 'is', as:

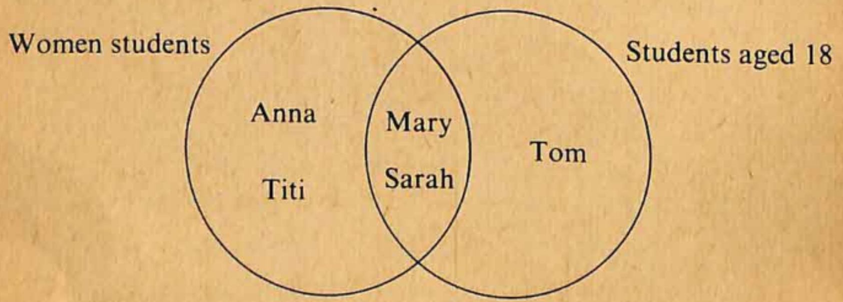
$$\{\text{Jacob, Anna, Ahmed, John}\} \cap \{\text{Jacob, Yusuf, Ahmed, John, Tom}\} = \{\text{Jacob, Ahmed, John}\}$$

Exercises

5. Show the set of men students and the set of students aged 20 on a Venn diagram. What is their intersection?
   Show the result using the symbol $\cap$.
6. Show {a, b, c, d} and {c, d, e, f, g} on a Venn diagram. What is their intersection?
   Show the result using the symbol $\cap$.
7. Show {2, 4, 6, 8} and {1, 2, 4, 8, 16} on a Venn diagram. What is their intersection?
   Show the result using the symbol $\cap$.

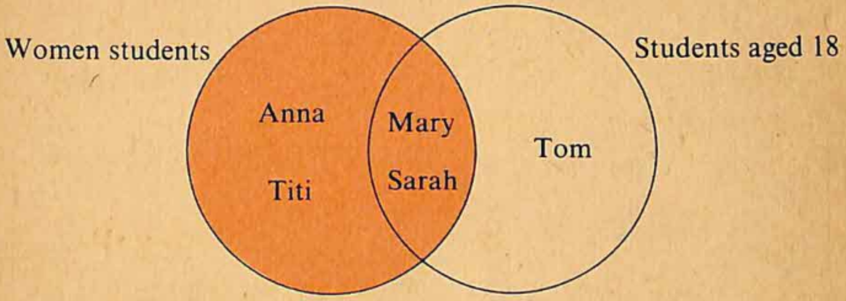
C. Union of sets

Here we show again the Venn diagram which we used on page 4. For ease of drawing we now use circles.

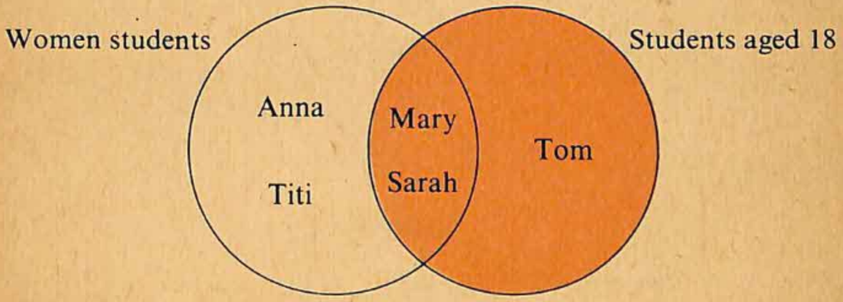


Looking at the diagram we see:

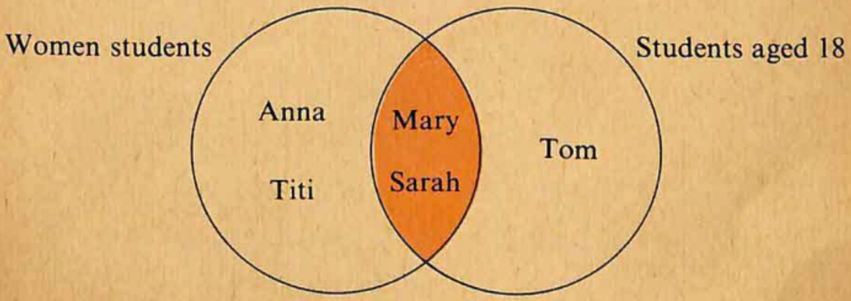
(a) The set of women students.



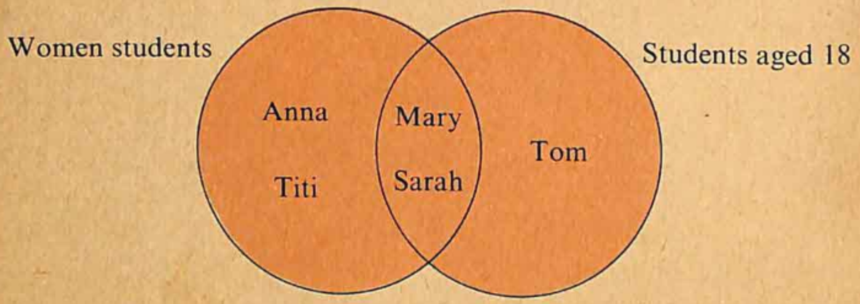
(b) The set of students aged 18.



(c) The set of women students who are aged 18.

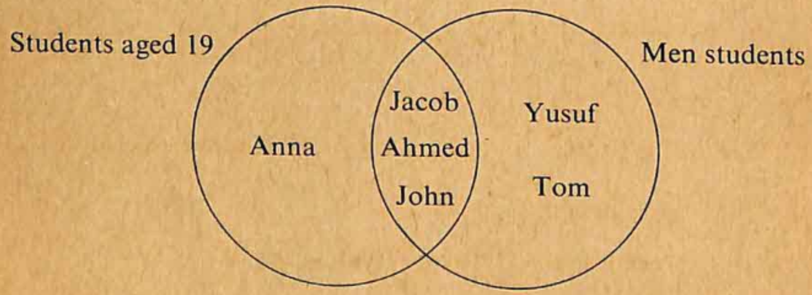


There is yet another set. It is {Anna, Titi, Mary, Sarah, Tom}. This contains all the women students and all the students aged 18.



We give this set a special name. We say that it is the union of the set of women students and the set of students aged 18.

Now we look again at the Venn diagram on page 5.



Here we see that the union of
{Anna, Jacob, Ahmed, John} and {Jacob, Ahmed, John, Yusuf, Tom}
is {Anna, Jacob, Ahmed, John, Yusuf, Tom}.

We write this more shortly, using the symbol ∪ for 'union', as:

{Anna, Jacob, Ahmed, John} ∪ {Jacob, Ahmed, John, Yusuf, Tom}
= {Anna, Jacob, Ahmed, John, Yusuf, Tom}

Here are three further examples of the union of sets:

$\{\text{Lagos, London, Moscow}\} \cup \{\text{Lagos, Paris, Cairo, London}\}$
$= \{\text{Lagos, London, Moscow, Paris, Cairo}\}$

$\{1, 3, 5, 9\} \cup \{3, 6, 9\} = \{1, 3, 5, 9, 6\}$

$\{p, x, w\} \cup \{m, n, p, w, v\} = \{p, x, w, m, n, v\}$

The symbols $\cap$ and $\cup$ may appear strange to us at first. However, a little practice in their use will help us to remember them.

Exercises

8. Show $\{e, n, g, l, i, s, h\}$ and $\{f, r, e, n, c, h\}$ on a Venn diagram. What is their union?
   Show the result using the symbol $\cup$.
9. Show $\{3, 6, 9, 12, 15\}$ and $\{4, 8, 12\}$ on a Venn diagram. What is their union?
   Show the result using the symbol $\cup$.
10. Show $\{2, 4, 6, 8, 10, 12, 14\}$ and $\{3, 6, 9, 12, 15\}$ on a Venn diagram. What is their union?
   Show the result using the symbol $\cup$.
11. Show $\{2, 4, 6, 8, 10\}$ and $\{1, 3, 5, 7, 9\}$ on a Venn diagram. What is their union?
   Show the result using the symbol $\cup$.
12. Show $\{g, o, a, t\}$ and $\{h, e, n\}$ on a Venn diagram. What is their union?
   Show the result using the symbol $\cup$.
13. What do you notice about the intersection of the sets in:
   (a) exercise 11,
   (b) exercise 12.

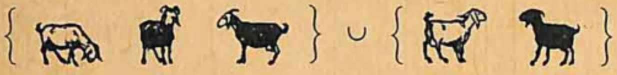
Important ideas in this chapter

1. The idea of a sub-set.
2. The use of Venn diagrams.
3. The idea of intersection; the use of $\cap$.
4. The idea of union; the use of $\cup$.

2. THE UNION OF DISJOINT SETS

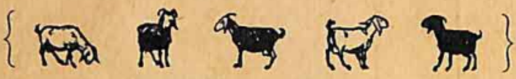
A. Disjoint sets

In chapter 1 we looked at the intersection and union of sets. We now go on to look at two more examples:



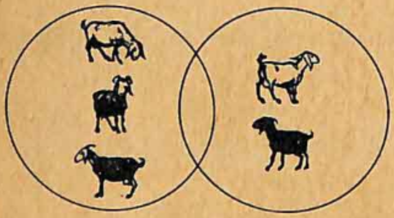
and $\{1, 3, 5, 7\} \cup \{2, 4, 6\}$.

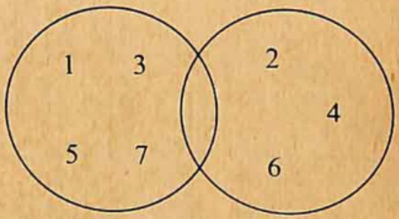
The result of the first union is



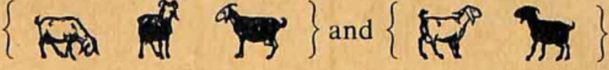
and of the second $\{1, 3, 5, 7, 2, 4, 6\}$.

We can use Venn diagrams to illustrate these examples.





We notice that in each case there is nothing in the intersection. In the first example no animal is in both



Sets such as these are called disjoint sets. In the second example no number is in both $\{1, 3, 5, 7\}$ and $\{2, 4, 6\}$. These are also disjoint sets.

Exercises

1. Use Venn diagrams to show the following pairs of sets:
   - (a) $\{A, L, I\}$ and $\{J, O, H, N\}$
   - (b) {lion, elephant, camel, giraffe} and {tiger, monkey, camel}
   - (c) {Anna, Tom, Mary, Jacob, John} and {Yusuf, Ahmed, Titi}
   - (d) {car, bicycle, bus} and {lorry, train, aeroplane, boat}
   - (e) {red, green} and {blue, red, orange, black, yellow}
   - (f) {France, Canada, Japan, Denmark} and {Sweden, Turkey, Japan, India}

   Which pairs of sets are disjoint?

2. Which of the following pairs of sets are disjoint?
   - (a) $\{a, b, c, d\}$ and $\{p, q, r, s, t\}$
   - (b) $\{l, m, n\}$ and $\{h, i, j, k, l\}$
   - (c) $\{1, 4, 9, 16\}$ and $\{2, 4, 8, 16\}$
   - (d) $\{10, 9, 8\}$ and $\{7, 6, 5, 4, 3, 2, 1\}$
   - (e) $\{J, A, C, O, B\}$ and $\{T, A, N, I\}$

3. In each of these examples the sets are disjoint. Copy and complete each statement.
   - (a) $\{a, b, c\} \cup \{d, e, f, g\} =$
   - (b) $\{p, q\} \cup \{l, m, n\} =$
   - (c) $\{a, e, i, o, u\} \cup \qquad\qquad = \{a, e, i, o, u, x, y, z\}$
   - (d) $\qquad\qquad \cup \{2, 4, 10\} = \{2, 4, 6, 8, 10\}$
   - (e) $\{98, 99\} \cup \{1, 3, 5\} =$
   - (f) $\{1, 4, 9, 16\} \cup \qquad\qquad = \{1, 4, 9, 16\}$

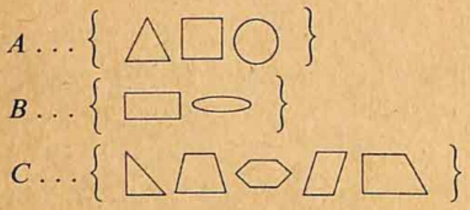
4. The union of $\{a, b, c\}$ and $\{d, e, f, g\}$ gives $\{a, b, c, d, e, f, g\}$. Write down other pairs of disjoint sets which, under union, give $\{a, b, c, d, e, f, g\}$.

5. Write down pairs of disjoint sets which, under union, give $\{2, 4, 6, 8\}$. Can you say how many pairs of such sets there are?

B. Union of more than two disjoint sets

Exercises

6. Look at these three sets:



Find:

(a) $A \cup B$ (b) $B \cup C$

Use the results to find:

(c) $(A \cup B) \cup C$ (d) $A \cup (B \cup C)$

The brackets are used to indicate which union is to be done first. What do you notice?

7. For the sets

$X \ldots \{1, 3, 5, 7\}$

$Y \ldots \{2, 4, 6\}$

$Z \ldots \{8\}$

find:

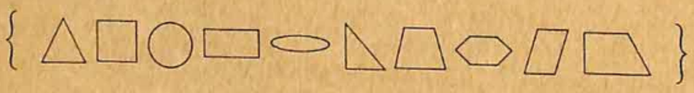
(a) $X \cup Y$ (b) $Y \cup Z$

Use the results to find:

(c) $(X \cup Y) \cup Z$ (d) $X \cup (Y \cup Z)$

What do you notice?

From exercise 6 we see that both $(A \cup B) \cup C$ and $A \cup (B \cup C)$ give



The result is the same whether we find $(A \cup B) \cup C$ or $A \cup (B \cup C)$. So we can write $(A \cup B) \cup C = A \cup (B \cup C)$. For this reason when we have three sets we often write $A \cup B \cup C$ to represent both $(A \cup B) \cup C$ and $A \cup (B \cup C)$. In the same way, from exercise 7, we see that $(X \cup Y) \cup Z = X \cup (Y \cup Z)$ and we use $X \cup Y \cup Z$ to represent both of them.

Exercises

8. Find:

   (a) $\{a, b\} \cup \{c, d\} \cup \{e, f\}$

   (b) $\{r, s, t\} \cup \{u, v, w\} \cup \{x, y, z\}$

   (c) $\{15, 16, 17, 18\} \cup \{10, 9, 8, 7\} \cup \{11, 12, 13, 14\}$

9. For the sets

   $A \ldots \{a, z\}$
   $B \ldots \{b, y\}$
   $C \ldots \{c, x\}$
   $D \ldots \{d, w\}$

   find:

   (a) $A \cup B \cup C$
   (b) $(A \cup B \cup C) \cup D$
   (c) $B \cup C \cup D$
   (d) $A \cup (B \cup C \cup D)$
   (e) $A \cup B \cup C \cup D$

Important ideas in this chapter

1. The union of two disjoint sets.
2. Union with more than two disjoint sets.

3. ADDITION AND SUBTRACTION

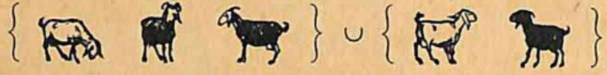
A. Looking again at the union of disjoint sets; the idea of addition

We begin this chapter by looking more closely at some disjoint sets and their union.

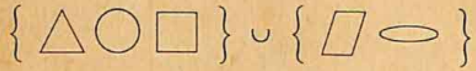
Exercises

1. Copy the two sets. Write their numbers underneath. Complete the union. Write the number of the resulting set underneath.

   (a) $\{\text{Mary, Ahmed, Sarah}\} \cup \{\text{Yusuf, Anna}\}$

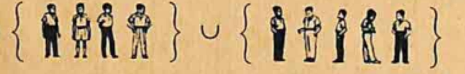
   (b) 

   (c) $\{a, b, c\} \cup \{d, e\}$

   (d) 

2. Copy the two sets. Write their numbers underneath. Complete the union. Write the number of the resulting set underneath.

   (a) $\{\text{Jacob, Anna, Mary, Tom}\} \cup \{\text{Yusuf, Sarah, John, Titi, Ahmed}\}$

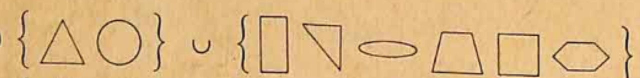
   (b) 

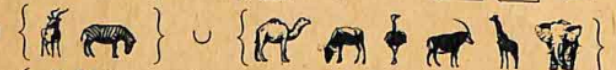
   (c) $\{l, m, n, o\} \cup \{p, q, r, s, t\}$

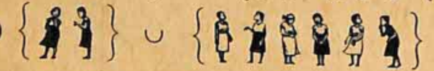
   (d) $\{\text{rice, cocoa, banana, orange}\} \cup \left\{\begin{matrix}\text{yam, palm-oil, chicken,} \\ \text{pepper, egg}\end{matrix}\right\}$

3. Copy the two sets. Write their numbers underneath. Complete the union. Write the number of the resulting set underneath.

   (a) $\{p, q\} \cup \{z, y, x, c, b, a\}$

   (b) 

   (c) 

   (d) 

From these exercises we see that

(a) The union of a set of number 3 with a set of number 2 gives a set of number 5.

(b) The union of a set of number 4 with a set of number 5 gives a set of number 9.

(c) The union of a set of number 2 with a set of number 6 gives a set of number 8.

These results are shown in the table below:

Number of first set	Number of second set	Number of set formed by union
3	2	5
4	5	9
2	6	8

Exercise

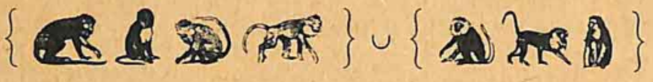
4. Copy this table. Illustrate each pair of numbers with sets as we did in exercises 1, 2, and 3. Complete the table.

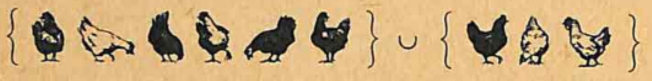
Number of first set	Number of second set	Number of set formed by union
2	4	
3	5	
7	2	
6	1	
8	0	

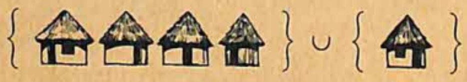
In each of the examples which we have done in this chapter we have formed the union of two disjoint sets. We wrote down the number of each of the original sets and then found the number of the set formed. This finding of the number of the set formed is given a special name. It is called the addition operation. We use the symbol + to indicate the operation and write, for example, $3 + 2$ and $4 + 5$. We show the results as $3 + 2 = 5$ and $4 + 5 = 9$. The symbol + is called the addition symbol. The number 5, for example, in $3 + 2 = 5$ is often called the sum of 3 and 2.

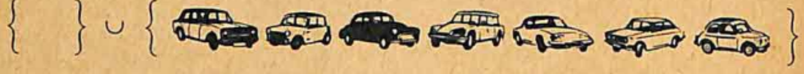
Exercise

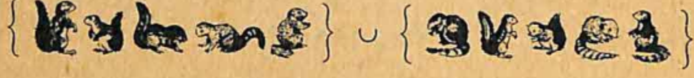
5. Complete the union. Write the corresponding addition.

(a) 

(b) 

(c) 

(d) 

(e) 

Each of the addition relationships such as $3 + 2 = 5$ and $4 + 3 = 7$ is called an addition fact. There are, of course, many of these facts. We need to memorize some of them if we wish to do our calculations quickly. However, because we use place-value we need to remember only the addition facts up to $9 + 9$. Here are some of these facts arranged in order.

$0 + 0 = 0$
$0 + 1 = 1$
$0 + 2 = 2$
$0 + 3 = 3$
$0 + 4 = 4$
$0 + 5 = 5$
$0 + 6 = 6$
$0 + 7 = 7$
$0 + 8 = 8$
$0 + 9 = 9$

$1 + 0 = 1$
$1 + 1 = 2$
$1 + 2 = 3$
$1 + 3 = 4$
$1 + 4 = 5$
$1 + 5 = 6$
$1 + 6 = 7$
$1 + 7 = 8$
$1 + 8 = 9$
$1 + 9 = 10$

$2 + 0 = 2$
$2 + 1 = 3$
$2 + 2 = 4$
$2 + 3 = 5$
etc.

Writing the facts in this way is slow and laborious. We often show them more concisely in another form. Here is a table which shows all the addition facts which we have met so far in this chapter:

		Second number									
	+	0	1	2	3	4	5	6	7	8	9
First number	0	0	1	2	3	4	5	6	7	8	9
	1	1	2	3	4	5	6	7	8	9	10
	2	2	3	4	5	6		8			
	3			5			8				
	4		5		7		9				
	5						10				
	6		7		9						
	7			9							
	8	8									
	9										

Exercise

6. Copy and complete the table above.

B. The idea of subtraction

Now that we have built up this set of addition facts we look at the kind of question each fact helps us to answer. Let us think about, for example, $3 + 2 = 5$. This addition fact enables us to answer the question 'What is the sum of three and two?'. It also gives us the answers to the questions 'What must be added to three to give five?' and 'What must be added to two to give five?'. We can show these questions with symbols as:

$$3 + 2 = \square$$
$$3 + \square = 5$$
$$\square + 2 = 5$$

where □ has to be replaced by a number.

The second and third of these questions are often written, in another way, using the − symbol. Instead of 3 + □ = 5 we write 5 − 3 = □, and instead of □ + 2 = 5 we write 5 − 2 = □. In examples such as these we know the number of one of the original sets and also the number of the set formed by the union. Finding the number of the other original set is called subtraction. The − symbol is called the subtraction symbol.

So far in this chapter we have used only *base ten*. Now, in some exercises, we use other bases.

Exercises

7. In this exercise we are working in *base five*.

(a) Copy and complete the addition:

(i) 1 + 2 =
(ii) 3 + 1 =
(iii) 2 + 0 =
(iv) 3 + 4 =
(v) 4 + 2 =
(vi) 0 + 3 =

(b) Copy and complete:

Second number

	+	0	1	2	3	4
First number	0					
	1					
	2					
	3					
	4					

(c) Use the table to complete:

(i) 1 + 3 =
(ii) 4 − 2 =
(iii) 0 + 2 =
(iv) 11 − 3 =
(v) 13 − 4 =
(vi) 2 + 4 =

8. In this exercise we are working in *base eight*.

(a) Copy and complete the addition:

(i) 5 + 3 =
(ii) 2 + 6 =
(iii) 5 + 4 =
(iv) 7 + 4 =
(v) 6 + 0 =
(vi) 7 + 7 =

(b) Copy and complete:

Second number

First number

+	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

(c) Use the table to complete:

(i) 6 + 1 =

(ii) 7 − 3 =

(iii) 11 − 4 =

(iv) 2 + 4 =

(v) 15 − 7 =

(vi) 7 + 6 =

9. In this exercise we are working in *base three*.

(a) Copy and complete the addition:

(i) 1 + 1 =

(ii) 2 + 0 =

(iii) 1 + 2 =

(iv) 0 + 1 =

(v) 2 + 2 =

(vi) 2 + 1 =

(b) Copy and complete:

Second number

First number

+	0	1	2
0			
1			
2			

(c) Use the table to complete:

(i) $1 + 2 =$	(iv) $11 - 2 =$
(ii) $2 - 1 =$	(v) $0 + 2 =$
(iii) $1 + 0 =$	(vi) $10 - 1 =$

C. Using addition and subtraction

We now look at some situations in which addition or subtraction is used.

There are five people in a room. Seven more come in. How many are now in the room? Here we are given the numbers of two sets of people. We have to find the number of the set formed by union. We use our knowledge of $5 + 7 = 12$ to say that there are now twelve people in the room.

All situations in which addition is used are similar to this. There are, however, several different ways in which subtraction is used:

(a) Mary has twelve oranges. She sells five of them. How many has she left? Here we see that the set of number 12 is formed by the union of the set Mary sells and the set she keeps. We have to find the number of the set she keeps. We show this as the subtraction $12 - 5$. Our knowledge of the addition fact $5 + 7 = 12$ enables us to say that she has seven oranges left.

(b) Mary has twelve oranges. Sarah has eight. How many more has Mary than Sarah? Here we see a somewhat different situation. We have to find the number of the set which under union with Sarah's set gives a set with the same number as Mary's. We show this as the subtraction $12 - 8$. Our knowledge of the addition fact $8 + 4 = 12$ enables us to say that Mary has four more than Sarah.

(c) The question in (b) could have been 'How many less has Sarah than Mary?' or 'What is the difference between the number of oranges Mary has and the number of oranges Sarah has?'. The answer to each of these is also found as in (b).

Important ideas in this chapter

1. The idea of addition.
2. Addition facts.
3. The idea of subtraction.
4. How we use addition and subtraction.

4. ADDITION, MULTIPLICATION, AND DIVISION

A. The addition of more than two numbers

So far we have been concerned with the addition of two numbers. Often, however, we are concerned with more than two numbers. For example, we may wish to add 2 and 3 and then add 4 to the result. We show this by writing

$$(2 + 3) + 4$$

The brackets are used to tell us which two numbers to add first. We show the working as:

$$\begin{aligned}(2 + 3) + 4 &= 5 + 4\\ &= 9\end{aligned}$$

Now we look at

$$2 + (3 + 4)$$

Here the brackets tell us to add the 3 and the 4 first. The complete working is set down as:

$$\begin{aligned}2 + (3 + 4) &= 2 + 7\\ &= 9\end{aligned}$$

We notice that the two answers are the same. That is,

$$(2 + 3) + 4 = 2 + (3 + 4)$$

For this reason we often omit the brackets and write $2 + 3 + 4$ to represent both $(2 + 3) + 4$ and $2 + (3 + 4)$.

When no brackets are put in we do, in fact, always work as if some are there. We need only to consider $1 + 99 + 17$ to see that in our minds most of us bracket the '$1 + 99$'. On the other hand, for $15 + 999 + 1$ we probably bracket the '$999 + 1$'.

Exercise

1. Copy and complete the following. Use brackets to show the order of your additions:

(a) $1 + 9 + 7$
(b) $9 + 1 + 99$
(c) $100 + 17 + 3$
(d) $2 + 8 + 17$
(e) $19 + 3 + 27$
(f) $2 + 18 + 70$

We have seen that additions such as, for example, $(2 + 3) + 4$ and $2 + (3 + 4)$ give the same answer. Both can be represented by $2 + 3 + 4$.

S.C.E.R.T., West Bengal
Date 21.3.75
517·521
S.C.E.R.T., West B
Library

We can, of course, extend this idea to additions such as 2 + 3 + 4 + 5, 5 + 7 + 4 + 8 + 9, etc.

B. The addition of equal numbers; the idea of multiplication

Exercise

2. Copy and complete:

(a) 3 + 3 + 3 + 3 + 3
(b) 2 + 2 + 2 + 2 + 2 + 2
(c) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
(d) 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7
(e) 9 + 9 + 9 + 9

To avoid writing the same number many times as in exercise 2, we introduce a short way of showing such additions. Instead of writing, for example,

3 + 3 + 3 + 3 + 3 = 15 we write 5 × 3 = 15
and for 9 + 9 + 9 + 9 = 36 we write 4 × 9 = 36

To help us to see that we are dealing with, for example, five *threes* we write the '5' first and the '3' second. Similarly for four nines we write the '4' first and the '9' second. With experience and practice we come to know that, for example, 5 × 3 = 15 without doing any addition. We then say that we are multiplying the five and the three. The operation is called multiplication and the × is called the multiplication symbol.

The statements 5 × 3 = 15 and 4 × 9 = 36 are both called multiplication facts. There are, of course, many of these facts. In order to calculate we need to memorize certain of them. But, because we use place-value, only those up to 9 × 9 need to be remembered. For convenience these facts are often set down in table form. For example, in *base ten* we have tables such as:

2 × 0 = 0
3 × 0 = 0
4 × 0 = 0
5 × 0 = 0
6 × 0 = 0
7 × 0 = 0
8 × 0 = 0
9 × 0 = 0

2 × 1 = 2
3 × 1 = 3
4 × 1 = 4
5 × 1 = 5
6 × 1 = 6
7 × 1 = 7
8 × 1 = 8
9 × 1 = 9

2 × 2 = 4
3 × 2 = 6
4 × 2 = 8
5 × 2 = 10
etc.

Sometimes these facts are shown in another table like this:

		Second number									
	×	0	1	2	3	4	5	6	7	8	9
First number	2	0	2	4							
	3	0	3	6							
	4	0	4	8							
	5	0	5	10							
	6	0	6								
	7	0	7								
	8	0	8								
	9	0	9								

Exercise

3. Copy and complete the table above.

From the table which we have just completed we see that, for example, 6 × 9 = 54 and 9 × 6 = 54. This tells us that, under multiplication, the pair of numbers (6, 9) gives 54 whichever of the two numbers is written first. For this reason we do not need to distinguish between 6 × 9 and 9 × 6; we do not have to worry about the order. In the same way 4 × 7 and 7 × 4, for example, have the same value. For this reason, if we know that, for example, 3 × 8 = 24 we can say straightaway that 8 × 3 = 24.

So far in our thinking about multiplication we have given a meaning to, for example, 6 × 1 and 6 × 0 but not to 1 × 6 and 0 × 6. Neither of these can be thought of in terms of repeated addition. However it is convenient to complete the patterns, such as,

$$\begin{aligned} 4 \times 6 &= 6 + 6 + 6 + 6 = 24 \\ 3 \times 6 &= 6 + 6 + 6 \qquad = 18 \\ 2 \times 6 &= 6 + 6 \qquad\qquad = 12 \end{aligned}$$

by saying that

$$1 \times 6 = 6$$

and also that

$$0 \times 6 = 0$$

With these extra facts in mind we can now extend our table of facts to read as:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercises

4. Write, in table form, the multiplication facts we need to learn when we work in:

   (a) *base five*
   (b) *base three*
   (c) *base six*
   (d) *base eight*
   (e) *base two*
   (f) *base four*

5. Complete another table of multiplication facts (*base ten*) for numbers up to, and including, 12. Colour all the squares in the main part of the table which contain the number 24. Do the same for the numbers 12, 36, and 10, using a different colour for each. What do you notice?

C. The idea of division

We start this section by looking at a multiplication fact, for example, $3 \times 7 = 21$. When we know this fact we can give the answer to various questions. We can use it to say what number must be put in place of the $\square$ in each of the following:

(a) $3 \times 7 = \square$
(b) $3 \times \square = 21$
(c) $\square \times 7 = 21$

The putting of 21 in place of the $\square$ in (a) is, of course, a direct use of the multiplication fact $3 \times 7 = 21$. In (b) and (c) however we are using the multiplication fact in a different way. Here we know one of a pair of numbers which, under multiplication, gives 21. We use the known fact to find the other number. This operation we call division.

There is no real need to introduce a new symbol to show division. In practice however it is convenient to use $\div$, and to write $21 \div 7 = \square$ instead of $7 \times \square = 21$. We read $21 \div 7$ as twenty-one divided by seven and find the answer by asking ourselves 'By what must I multiply 7 to get 21?'. Our knowledge of the multiplication fact $7 \times 3 = 21$ enables us to give the answer 3.

Exercises

6. Rewrite, using the division sign:

(a) 3 × □ = 12
(b) 4 × □ = 36
(c) □ × 5 = 20
(d) 9 × □ = 72
(e) □ × 8 = 56
(f) □ × 7 = 49
(g) 5 × □ = 45
(h) □ × 2 = 8
(i) 9 × □ = 81

7. Write as a multiplication:

(a) 36 ÷ 9 = □
(b) 54 ÷ 6 = □
(c) 42 ÷ 7 = □
(d) 24 ÷ 8 = □
(e) 27 ÷ 3 = □
(f) 63 ÷ 7 = □
(g) 32 ÷ 8 = □
(h) 16 ÷ 4 = □
(i) 56 ÷ 7 = □

8. Use the tables of multiplication facts in exercise 4 to find the answer to the division:

(a) 22 ÷ 4 (*base five*)
(b) 11 ÷ 2 (*base three*)
(c) 32 ÷ 5 (*base six*)
(d) 25 ÷ 3 (*base eight*)
(e) 30 ÷ 4 (*base eight*)
(f) 12 ÷ 2 (*base four*)

D. Using multiplication and division

We now look at some situations in which multiplication or division is used:

(a) Three boxes of tins of food are delivered to a shop. Each box contains six tins. How many tins are there altogether? Here we have to give the answer to 3 × 6. We use our knowledge of the fact 3 × 6 = 18 to say there are eighteen tins altogether.

(b) The owner of the shop decides to order some more tins. He wants another thirty. How many boxes will he order? Here we have to give the answer to 30 ÷ 6. We use our knowledge of the fact 6 × 5 = 30 and say 30 ÷ 6 = 5. He must order another five boxes.

(c) The suppliers decide to alter the size of their boxes. When the owner of the shop orders more tins he finds that in seven boxes there are now altogether fifty-six tins. How many are there in each box? Here we have to give the answer to 56 ÷ 7. We use our knowledge of the fact 7 × 8 = 56 and say 56 ÷ 7 = 8. There are now eight tins in each box.

Exercises

In each of these exercises write the multiplication or division which has to be answered. Give the multiplication fact which is used and also the answer to the problem.

9. There are twenty-four people going to a wedding. How many cars will they need if each car carries six people?
10. Tom has a heap of oranges. In it there are forty-two oranges. He shares them with six of his friends. How many oranges will each get?
11. John buys nine bags of meal. Each bag contains four pounds of meal. How many pounds of meal does he buy?
12. A class of thirty children is to be arranged in teams of five. How many teams will there be?
13. A class of thirty children is to be arranged in five teams. How many children will there be in each team?

Important ideas in this chapter

1. The addition of more than two numbers.
2. The idea of multiplication.
3. The idea of division.
4. How we use multiplication and division.

S.C.E.R.T., West Bengal
Library
Calcutta
25/3, B. C. Road

5. MORE ABOUT SETS AND NUMBERS

A. Operations

So far in this book we have been looking at:

(a) the union and intersection of sets

(b) the addition, subtraction, multiplication, and division of numbers.

In each case we have been doing something with two sets or two numbers. For example, we have found the union of two sets. We have also multiplied two numbers. Because something is being done with the sets or numbers we say that we operate on the sets or numbers. We call the various things we do operations and use phrases such as the addition operation, the division operation, the operation of union.

We now go on to look more closely at some of the properties of sets and numbers under the various operations.

B. The commutative property

Exercises

1. Copy and complete (*base ten*):

(a) $5 + 2 =$
(b) $2 + 5 =$
(c) $7 + 4 =$
(d) $4 + 7 =$
(e) $1 + 99 =$
(f) $99 + 1 =$
(g) $11 + 0 =$
(h) $0 + 11 =$

What do you notice?

2. Copy and complete (*base five*):

(a) $1 + 2 =$
(b) $2 + 1 =$
(c) $4 + 1 =$
(d) $1 + 4 =$
(e) $3 + 0 =$
(f) $0 + 3 =$
(g) $4 + 2 =$
(h) $2 + 4 =$

What do you notice?

3. Which of the following statements (*base ten*) are true?

(a) $5 + 4 = 4 + 5$
(b) $6 - 2 = 2 - 6$
(c) $8 - 5 = 5 - 8$
(d) $4 + 6 = 6 + 4$
(e) $7 + 2 = 2 + 7$
(f) $9 - 1 = 1 - 9$
(g) $6 + 0 = 0 + 6$
(h) $5 - 0 = 0 - 5$

What do you notice?

4. Copy and complete (*base ten*):

   (a) $7 \times 4 =$
   (b) $4 \times 7 =$
   (c) $1 \times 9 =$
   (d) $9 \times 1 =$
   (e) $7 \times 8 =$
   (f) $8 \times 7 =$
   (g) $5 \times 0 =$
   (h) $0 \times 5 =$

   What do you notice?

5. Copy and complete (*base eight*):

   (a) $5 \times 3 =$
   (b) $3 \times 5 =$
   (c) $4 \times 0 =$
   (d) $0 \times 4 =$
   (e) $6 \times 7 =$
   (f) $7 \times 6 =$
   (g) $2 \times 1 =$
   (h) $1 \times 2 =$

   What do you notice?

6. Which of the following statements (*base ten*) are true?

   (a) $6 \times 5 = 5 \times 6$
   (b) $7 \times 8 = 8 \times 7$
   (c) $6 \div 2 = 2 \div 6$
   (d) $12 \div 3 = 3 \div 12$
   (e) $15 \div 5 = 5 \div 15$
   (f) $4 \times 0 = 0 \times 4$
   (g) $6 \times 1 = 1 \times 6$
   (h) $6 \div 1 = 1 \div 6$

   What do you notice?

7. Write, in words, the conclusions you have reached from exercises 1 to 6.

In doing these exercises we have been looking at the effects of changing the order of a pair of numbers under the various operations. From them we see that for addition and multiplication a change of order does not alter the result. In subtraction and division, however, a change of order does alter the result, except in one or two special examples.

When a change of order does not alter the result we say that the operation is commutative. We say that for addition and multiplication the natural numbers have the commutative property. For subtraction and division the natural numbers do not have the commutative property.

Exercises

8. For the sets P and Q, where

   P is {a, b, c, d}
   Q is {a, b, e, f, g}

find:

(a) $P \cup Q$ (c) $P \cap Q$

(b) $Q \cup P$ (d) $Q \cap P$

What do you notice?

9. For the sets R and S, where

R is $\{2, 4, 6, 8, 10\}$

S is $\{1, 2, 4, 8\}$

find:

(a) $R \cup S$ (c) $R \cap S$

(b) $S \cup R$ (d) $S \cap R$

What do you notice?

10. Do you think that sets under (a) union, (b) intersection, have the commutative property? Illustrate by using Venn diagrams.

C. The associative property

Exercises

11. Copy and complete (*base ten*):

(a) $(4 + 5) + 2 =$ (e) $(6 + 0) + 9 =$

(b) $4 + (5 + 2) =$ (f) $6 + (0 + 9) =$

(c) $7 + (1 + 3) =$ (g) $(2 + 8) + 7 =$

(d) $(7 + 1) + 3 =$ (h) $2 + (8 + 7) =$

What do you notice?

12. Copy and complete (*base five*):

(a) $(2 + 4) + 3 =$ (e) $4 + (1 + 4) =$

(b) $2 + (4 + 3) =$ (f) $(4 + 1) + 4 =$

(c) $1 + (2 + 3) =$ (g) $3 + (0 + 2) =$

(d) $(1 + 2) + 3 =$ (h) $(3 + 0) + 2 =$

What do you notice?

13. Which of the following statements (*base ten*) are true?

(a) $(5 + 4) + 1 = 5 + (4 + 1)$

(b) $(6 - 2) - 1 = 6 - (2 - 1)$

(c) $8 - (5 - 3) = (8 - 5) - 3$

(d) $8 + (2 + 7) = (8 + 2) + 7$

What do you notice?

14. Copy and complete (*base ten*):

(a) $(3 \times 2) \times 4 =$
(b) $3 \times (2 \times 4) =$
(c) $5 \times (2 \times 6) =$
(d) $(5 \times 2) \times 6 =$
(e) $(6 \times 1) \times 9 =$
(f) $6 \times (1 \times 9) =$
(g) $8 \times (0 \times 5) =$
(h) $(8 \times 0) \times 5 =$

What do you notice?

15. Copy and complete (*base eight*):

(a) $(3 \times 2) \times 4 =$
(b) $3 \times (2 \times 4) =$
(c) $3 \times (1 \times 5) =$
(d) $(3 \times 1) \times 5 =$
(e) $(7 \times 0) \times 2 =$
(f) $7 \times (0 \times 2) =$
(g) $6 \times (1 \times 5) =$
(h) $(6 \times 1) \times 5 =$

What do you notice?

16. Which of the following statements (*base ten*) are true?

(a) $(4 \times 3) \times 2 = 4 \times (3 \times 2)$
(b) $(24 \div 6) \div 2 = 24 \div (6 \div 2)$
(c) $7 \times (3 \times 4) = (7 \times 3) \times 4$
(d) $27 \div (9 \div 3) = (27 \div 9) \div 3$

What do you notice?

From these exercises we see that, for example,

$$(4 + 5) + 2 = 4 + (5 + 2)$$
$$(3 \times 2) \times 4 = 3 \times (2 \times 4)$$
but $(8 - 5) - 3 \neq 8 - (5 - 3)$
$$(24 \div 6) \div 2 \neq 24 \div (6 \div 2).$$

Note: The symbol $\neq$ is read as 'is not equal to'.

Changing the position of the brackets for addition and multiplication does not alter the result. In subtraction and division however it usually does.

When the result is not changed we say that the operation is associative. For the operations addition and multiplication the natural numbers have the associative property. For subtraction and division the natural numbers do not have the associative property.

Exercises

17. For the sets A, B, and C where

A is $\{a, b, c, d\}$

B is $\{a, b, c, e, f\}$

C is $\{a, b, c, g, h, k\}$

find:

(a) $(A \cup B) \cup C$

(b) $A \cup (B \cup C)$

(c) $(A \cap B) \cap C$

(d) $A \cap (B \cap C)$

What do you notice?

18. For the sets P, Q, and R where

P is $\{2, 4, 6, 8, 10\}$

Q is $\{3, 6, 9, 12\}$

R is $\{6, 12, 18\}$

find:

(a) $(P \cup Q) \cup R$

(b) $P \cup (Q \cup R)$

(c) $(P \cap Q) \cap R$

(d) $P \cap (Q \cap R)$

What do you notice?

19. Do you think that sets under (a) union, (b) intersection, have the associative property? Illustrate by using Venn diagrams.

D. The distributive property

Exercises

20. Find the values (*base ten*) of:

(a) $2 \times (5 + 3)$ and $(2 \times 5) + (2 \times 3)$

(b) $5 \times (6 + 2)$ and $(5 \times 6) + (5 \times 2)$

(c) $4 \times (3 + 0)$ and $(4 \times 3) + (4 \times 0)$

(d) $6 \times (5 + 1)$ and $(6 \times 5) + (6 \times 1)$

What do you notice?

21. Find the values (*base five*) of:

(a) $2 \times (1 + 3)$ and $(2 \times 1) + (2 \times 3)$

(b) $4 \times (2 + 1)$ and $(4 \times 2) + (4 \times 1)$

(c) $3 \times (0 + 2)$ and $(3 \times 0) + (3 \times 2)$

(d) $2 \times (3 + 2)$ and $(2 \times 3) + (2 \times 2)$

What do you notice?

From these exercises we see that, for example,

$$2 \times (5 + 3) = (2 \times 5) + (2 \times 3).$$

Likewise $$5 \times (6 + 2) = (5 \times 6) + (5 \times 2), \text{ etc.}$$

This kind of rewriting can be done for any three natural numbers. We describe this property by saying that, for the natural numbers, multiplication is distributive over addition.

Exercises

22. Find the values (*base ten*) of:

    (a) $2 \times (4 - 1)$ and $(2 \times 4) - (2 \times 1)$
    (b) $3 \times (3 - 2)$ and $(3 \times 3) - (3 \times 2)$
    (c) $2 \times (5 - 2)$ and $(2 \times 5) - (2 \times 2)$
    (d) $3 \times (6 - 3)$ and $(3 \times 6) - (3 \times 3)$

    Is it true to say that multiplication is distributive over subtraction for the natural numbers?

23. Make up some examples to test whether, for the natural numbers:

    (a) Subtraction is distributive over multiplication.
    (b) Division is distributive over addition.
    (c) Addition is distributive over multiplication.

E. Nought and one

We now look more closely at two numbers which have special properties.

Exercises

24. Copy and complete:

(a) *base ten:* (i) 7 + 0 =
(ii) 0 + 5 =
(iii) 18 + 0 =

(b) *base five:* (i) 0 + 4 =
(ii) 3 + 0 =
(iii) 0 + 10 =

(c) *base three:* (i) 2 + 0 =
(ii) 0 + 1 =
(iii) 12 + 0 =

25. Copy and complete:

(a) *base ten:* (i) $7 \times 1 =$
(ii) $1 \times 5 =$
(iii) $18 \times 1 =$

(b) *base five:* (i) $1 \times 4 =$
(ii) $3 \times 1 =$
(iii) $1 \times 10 =$

(c) *base three:* (i) $2 \times 1 =$
(ii) $1 \times 1 =$
(iii) $12 \times 1 =$

From these exercises we see that:

(a) The addition of any natural number and nought gives the original number. Nought is the only natural number which has this property.

(b) The multiplication of any natural number and one gives the original number. One is the only natural number which has this property.

Numbers such as nought and one which, for certain operations, leave a number unchanged become increasingly important as we extend our knowledge of mathematics. A special name is given to them. Each is called an identity element. (*Note:* the members of a set are often called the elements of the set.)

We end this chapter by looking at multiplication and division by nought.

Exercise

26. Copy and complete, where possible:

(a) *base ten:* (i) $7 \times 0 =$
(ii) $0 \times 5 =$
(iii) $0 \div 4 =$
(iv) $3 \div 0 =$

(b) *base five:* (i) $0 \times 4 =$
(ii) $3 \times 0 =$
(iii) $0 \div 2 =$
(iv) $1 \div 0 =$

From these exercises we see that:

(a) The multiplication of any natural number and nought gives nought (e.g. $3 \times 0 = 0$, $0 \times 3 = 0$).

(b) The division of nought by any other natural number gives nought (e.g. $0 \div 3 = 0$).

(c) The division of any natural number by nought (e.g. $3 \div 0$, $0 \div 0$) presents a problem. This we must look at more closely.

Let us consider $3 \div 0$. Here we have to find a natural number to replace $\square$ in

$$0 \times \square = 3$$

We know, however, that the multiplication of 0 and any other number gives 0. There is no number which can be put in place of $\square$ in $0 \times \square = 3$. Similarly there is no number which we can put in place of $\square$ in, for example, $0 \times \square = 7$, $0 \times \square = 4$, etc. So $3 \div 0$, $7 \div 0$, $4 \div 0$, etc. have no answers. In fact, when we try to divide any natural number, except 0, by 0 there is no answer.

Dividing 0 by 0 leads us to a different situation. Here we have to find a natural number to replace $\square$ in

$$0 \times \square = 0$$

We know, however, that the multiplication of 0 and *any other number* gives 0. We can put any number in place of $\square$. This tells us that $0 \div 0$ has many answers. There is no particular natural number which we can give as the answer.

We are now familiar with the properties of the natural numbers under the usual operations and are aware of the special properties of 0 and 1. We go on to use these ideas in Book 5, *Working with Numbers.*

Important ideas in this chapter

1. The idea of an operation.
2. The commutative property.
3. The associative property.
4. The distributive property.
5. The idea of nought and one as identity elements.

ANSWERS

1. UNION OF SETS

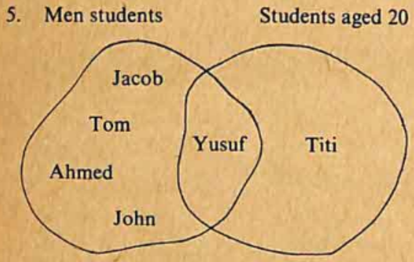
1. (a) {Mary, Anna, Sarah, Titi}
   (b) {Jacob, Anna, Yusuf, Ahmed, John, Titi}
   (c) {Mary, Anna, Ahmed, John}
   (d) {Mary, Sarah, Tom}
   (e) {Jacob, Yusuf, Sarah, Titi, Tom}

2. (a) {January, June, July}
   (b) {September, October, November, December}
   (c) {January, March, May, July, August, October, December}
   (d) {April, June, September, November}
   (e) { }

3. (a) {6, 7, 8, 9}
   (b) {1, 2, 3, 4}
   (c) {2, 4, 6, 8}
   (d) {1, 3, 5, 7, 9}
   (e) {4, 5, 6, 7}

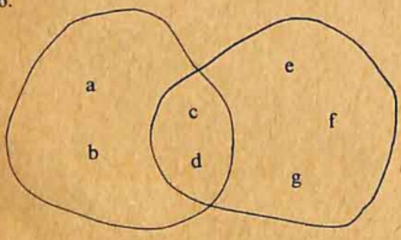
4. (a) {e, u, a, i, o}
   (b) {d, c, t, n}

5.



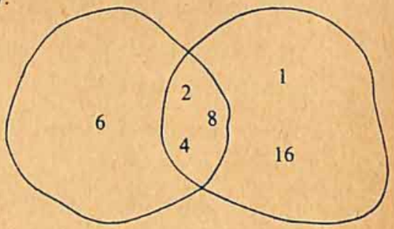
{Jacob, Yusuf, Ahmed, John, Tom} ∩ {Yusuf, Titi} = {Yusuf}

6.



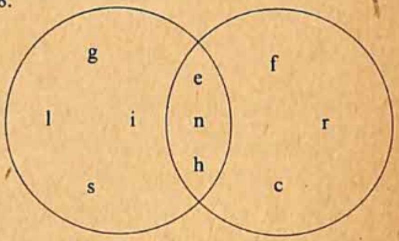
{a, b, c, d} ∩ {c, d, e, f, g} = {c, d}

7.



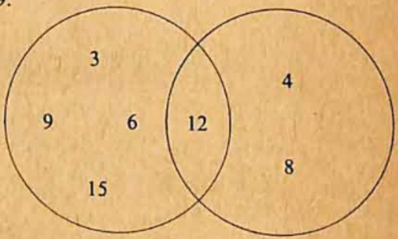
{2, 4, 6, 8} ∩ {1, 2, 4, 8, 16} = {2, 4, 8}

8.



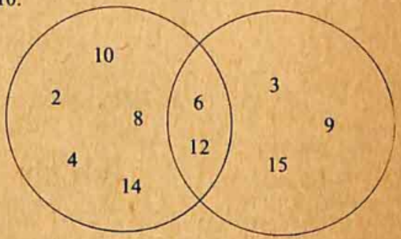
{e, n, g, l, i, s, h} ∪ {f, r, e, n, c, h} = {e, n, g, l, i, s, h, f, r, c}

9.



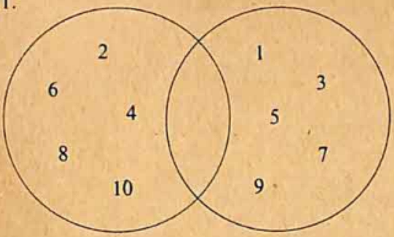
{3, 6, 9, 12, 15} ∪ {4, 8, 12} = {3, 6, 9, 12, 15, 4, 8}

10.



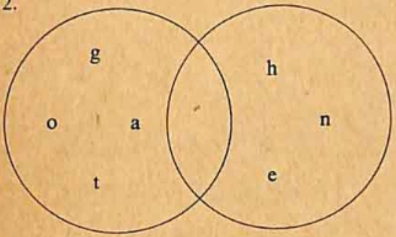
{2, 4, 6, 8, 10, 12, 14} ∪ {3, 6, 9, 12, 15} = {2, 4, 6, 8, 10, 12, 14, 3, 9, 15}

11.



{2, 4, 6, 8, 10} ∪ {1, 3, 5, 7, 9} =
{2, 4, 6, 8, 10, 1, 3, 5, 7, 9}

12.



{g, o, a, t} ∪ {h, e, n} = {g, o, a, t, h, e, n}

13. (a) The intersection is { }.
(b) The intersection is { }.

2. THE UNION OF DISJOINT SETS

1. (a), (c), (d).

2. (a), (d).

3. (a) {a, b, c} ∪ {d, e, f, g} = {a, b, c, d, e, f, g}
(b) {p, q} ∪ {l, m, n} = {p, q, l, m, n}
(c) {a, e, i, o, u} ∪ {x, y, z} =
{a, e, i, o, u, x, y, z}
(d) {6, 8} ∪ {2, 4, 10} = {2, 4, 6, 8, 10}
(e) {98, 99} ∪ {1, 3, 5} = {98, 99, 1, 3, 5}
(f) {1, 4, 9, 16} ∪ { } = {1, 4, 9, 16}

4. { }, {a, b, c, d, e, f, g};
{a}, {b, c, d, e, f, g};
{d, e, g}, {a, b, c, f}
These are only a few of the possible pairs of disjoint sets. There are many others.

5. { }, {2, 4, 6, 8};
{2}, {4, 6, 8};
{4, 8}, {2, 6}.
These are three of the possible pairs of disjoint sets. There are eight pairs.

6. (a) { △ □ ○ ▭ ⬭ }
(b) { ▭ ⬭ ◺ ⏢ ⬭ ▱ ◸ }
(c) { △ □ ○ ▭ ⬭ ◺ ⏢ ⬭ ▱ ◸ }
(d) { △ □ ○ ▭ ⬭ ◺ ⏢ ⬭ ▱ ◸ }

$(A \cup B) \cup C = A \cup (B \cup C)$

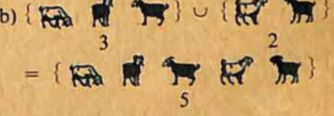
7. (a) {1, 3, 5, 7, 2, 4, 6}
(b) {2, 4, 6, 8}
(c) {1, 3, 5, 7, 2, 4, 6, 8}
(d) {1, 3, 5, 7, 2, 4, 6, 8}
$(X \cup Y) \cup Z = X \cup (Y \cup Z)$

8. (a) {a, b, c, d, e, f}
(b) {r, s, t, u, v, w, x, y, z}
(c) {15, 16, 17, 18, 10, 9, 8, 7, 11, 12, 13, 14}

9. (a) {a, z, b, y, c, x}
(b) {a, z, b, y, c, x, d, w}
(c) {b, y, c, x, d, w}
(d) {a, z, b, y, c, x, d, w}
(e) {a, z, b, y, c, x, d, w}

3. ADDITION AND SUBTRACTION

1. (a) {Mary, Ahmed, Sarah} ∪ {Yusuf, Anna}
3 2
= {Mary, Ahmed, Sarah, Yusuf, Anna}
5

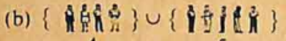
(b)


(c) {a, b, c} ∪ {d, e} = {a, b, c, d, e}
3 2 5

ANSWERS

(d) { △□○ } ∪ { ▱⬭ }
3 2
= { △□○▱⬭ }
5

2. (a) {Jacob, Anna, Mary, Tom} ∪ {Yusuf, Sarah, John, Titi, Ahmed}
4 5
= {Jacob, Anna, Mary, Tom, Yusuf, Sarah, John, Titi, Ahmed}
9

(b)

4 5

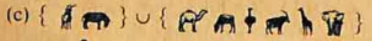
9

(c) {l, m, n, o} ∪ {p, q, r, s, t}
4 5
= {l, m, n, o, p, q, r, s, t}
9

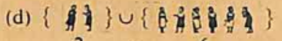
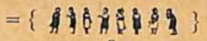
(d) {rice, cocoa, banana, orange}
4
∪ {yam, palm-oil, egg, chicken, pepper}
5
= {rice, cocoa, banana, orange, yam, palm-oil, egg, chicken, pepper}
9

3. (a) {p, q} ∪ {z, y, x, c, b, a}
2 6
= {p, q, z, y, x, c, b, a}
8

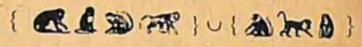
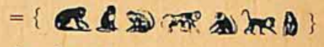
(b) { △○ } ∪ { ▯◸⬭⏢□⬭ }
2 6
= { △○▯◸⬭⏢□⬭ }
8

(c)

2 6

8

(d)

2 6

8

4. Number of first set	Number of second set	Number of set formed by union
2	4	6
3	5	8
7	2	9
6	1	7
8	0	8

5. (a)


4 + 3 = 7

(b)

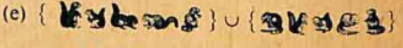
6 + 3 = 9

(c)

4 + 1 = 5

(d)

0 + 7 = 7

(e)


5 + 5 = 10

6. Second number

First number

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

7. (a) (i) $1 + 2 = 3$
(ii) $3 + 1 = 4$
(iii) $2 + 0 = 2$
(iv) $3 + 4 = 12$
(v) $4 + 2 = 11$
(vi) $0 + 3 = 3$

(b) Second number

First number

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

(c) (i) $3 + 1 = 4$
(ii) $4 - 2 = 2$
(iii) $2 + 0 = 2$
(iv) $11 - 3 = 3$
(v) $13 - 4 = 4$
(vi) $2 + 4 = 11$

8. (a) (i) $5 + 3 = 10$
(ii) $2 + 6 = 10$
(iii) $5 + 4 = 11$
(iv) $7 + 4 = 13$
(v) $6 + 0 = 6$
(vi) $7 + 7 = 16$

(b) Second number

First number

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

(c) (i) $6 + 1 = 7$
(ii) $7 - 3 = 4$
(iii) $11 - 4 = 5$
(iv) $2 + 4 = 6$
(v) $15 - 7 = 6$
(vi) $7 + 6 = 15$

9. (a) (i) $1 + 1 = 2$
(ii) $2 + 0 = 2$
(iii) $1 + 2 = 10$
(iv) $0 + 1 = 1$
(v) $2 + 2 = 11$
(vi) $2 + 1 = 10$

(b) Second number

First number

+	0	1	2
0	0	1	2
1	1	2	10
2	2	10	11

(c) (i) $1 + 2 = 10$
(ii) $2 - 1 = 1$
(iii) $1 + 0 = 1$
(iv) $11 - 2 = 2$
(v) $0 + 2 = 2$
(vi) $10 - 1 = 2$

4. ADDITION, MULTIPLICATION, AND DIVISION

1. (a)
$$\begin{aligned}1 + 9 + 7 &= (1 + 9) + 7\\ &= 10 + 7\\ &= 17\end{aligned}$$

(b)
$$\begin{aligned}9 + 1 + 99 &= 9 + (1 + 99)\\ &= 9 + 100\\ &= 109\end{aligned}$$

(c)
$$\begin{aligned}100 + 17 + 3 &= 100 + (17 + 3)\\ &= 100 + 20\\ &= 120\end{aligned}$$

(d)
$$\begin{aligned}2 + 8 + 17 &= (2 + 8) + 17\\ &= 10 + 17\\ &= 27\end{aligned}$$

(e)
$$\begin{aligned}19 + 3 + 27 &= 19 + (3 + 27)\\ &= 19 + 30\\ &= 49\end{aligned}$$

(f)
$$\begin{aligned}2 + 18 + 70 &= (2 + 18) + 70\\ &= 20 + 70\\ &= 90\end{aligned}$$

There are, of course, other ways of using the brackets.

2. (a) $3 + 3 + 3 + 3 + 3 = 15$
(b) $2 + 2 + 2 + 2 + 2 + 2 = 12$
(c) $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45$
(d) $7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 = 56$
(e) $9 + 9 + 9 + 9 = 36$

ANSWERS

3\.

Second number

First number

×	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

4. (a)

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

(b)

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	11

(c)

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	10	13	20	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

(d)

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

(e)

×	0	1
0	0	0
1	0	1

(f)

×	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

5\.

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

6. (a) 12 ÷ 3 = ☐
(b) 36 ÷ 4 = ☐
(c) 20 ÷ 5 = ☐
(d) 72 ÷ 9 = ☐
(e) 56 ÷ 8 = ☐
(f) 49 ÷ 7 = ☐
(g) 45 ÷ 5 = ☐
(h) 8 ÷ 2 = ☐
(i) 81 ÷ 9 = ☐

7. (a) 9 × ☐ = 36
or ☐ × 9 = 36

(b) 6 × ☐ = 54
or ☐ × 6 = 54

(c) 7 × ☐ = 42
or ☐ × 7 = 42

(d) 8 × ☐ = 24
or ☐ × 8 = 24

(e) 3 × ☐ = 27
or ☐ × 3 = 27

(f) 7 × ☐ = 63
or ☐ × 7 = 63

(g) 8 × ☐ = 32
or ☐ × 8 = 32

(h) 4 × ☐ = 16
or ☐ × 4 = 16

(i) 7 × ☐ = 56
or ☐ × 7 = 56

8. (a) 3
(b) 2
(c) 4
(d) 7
(e) 6
(f) 3

9. 4

10. 6

11. 36

12. 6

13. 6

5. MORE ABOUT SETS AND NUMBERS

1. (a) 5 + 2 = 7
(b) 2 + 5 = 7
(c) 7 + 4 = 11
(d) 4 + 7 = 11
(e) 1 + 99 = 100
(f) 99 + 1 = 100
(g) 11 + 0 = 11
(h) 0 + 11 = 11

2. (a) 1 + 2 = 3
(b) 2 + 1 = 3
(c) 4 + 1 = 10
(d) 1 + 4 = 10
(e) 3 + 0 = 3
(f) 0 + 3 = 3
(g) 4 + 2 = 11
(h) 2 + 4 = 11

3. (a), (d), (e), and (g).

4. (a) 7 × 4 = 28
(b) 4 × 7 = 28
(c) 1 × 9 = 9
(d) 9 × 1 = 9
(e) 7 × 8 = 56
(f) 8 × 7 = 56
(g) 5 × 0 = 0
(h) 0 × 5 = 0

5. (a) 5 × 3 = 17
(b) 3 × 5 = 17
(c) 4 × 0 = 0
(d) 0 × 4 = 0
(e) 6 × 7 = 52
(f) 7 × 6 = 52
(g) 2 × 1 = 2
(h) 1 × 2 = 2

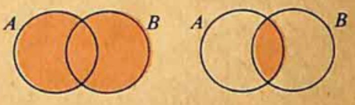
6. (a), (b), (f), and (g).

7. If 'a' and 'b' are two numbers, then it is always true that a + b = b + a
and a × b = b × a.
But (a − b) is not equal to (b − a)
and (a ÷ b) is not equal to (b ÷ a)
except in the special case when a = b.

8. (a) $P \cup Q = \{a, b, c, d, e, f, g\}$
(b) $Q \cup P = \{a, b, c, d, e, f, g\}$
$P \cup Q = Q \cup P$
(c) $P \cap Q = \{a, b\}$
(d) $Q \cap P = \{a, b\}$
$P \cap Q = Q \cap P$

9. (a) $R \cup S = \{1, 2, 4, 6, 8, 10\}$
(b) $S \cup R = \{1, 2, 4, 6, 8, 10\}$
$R \cup S = S \cup R$
(c) $R \cap S = \{2, 4, 8\}$
(d) $S \cap R = \{2, 4, 8\}$
$R \cap S = S \cap R$

10. (a) Yes. (b) Yes.



$A \cup B$ and $B \cup A$ $A \cap B$ and $B \cap A$

11. (a) (4 + 5) + 2 = 9 + 2 = 11
(b) 4 + (5 + 2) = 4 + 7 = 11
(c) 7 + (1 + 3) = 7 + 4 = 11
(d) (7 + 1) + 3 = 8 + 3 = 11
(e) (6 + 0) + 9 = 6 + 9 = 15
(f) 6 + (0 + 9) = 6 + 9 = 15
(g) (2 + 8) + 7 = 10 + 7 = 17
(h) 2 + (8 + 7) = 2 + 15 = 17

12. (a) (2 + 4) + 3 = 11 + 3 = 14
(b) 2 + (4 + 3) = 2 + 12 = 14
(c) 1 + (2 + 3) = 1 + 10 = 11
(d) (1 + 2) + 3 = 3 + 3 = 11
(e) 4 + (1 + 4) = 4 + 10 = 14
(f) (4 + 1) + 4 = 10 + 4 = 14
(g) 3 + (0 + 2) = 3 + 2 = 10
(h) (3 + 0) + 2 = 3 + 2 = 10

13. (a) and (d).

14. (a) (3 × 2) × 4 = 6 × 4 = 24
(b) 3 × (2 × 4) = 3 × 8 = 24
(c) 5 × (2 × 6) = 5 × 12 = 60
(d) (5 × 2) × 6 = 10 × 6 = 60
(e) (6 × 1) × 9 = 6 × 9 = 54
(f) 6 × (1 × 9) = 6 × 9 = 54
(g) 8 × (0 × 5) = 8 × 0 = 0
(h) (8 × 0) × 5 = 0 × 5 = 0

15. (a) (3 × 2) × 4 = 6 × 4 = 30
(b) 3 × (2 × 4) = 3 × 10 = 30
(c) 3 × (1 × 5) = 3 × 5 = 17
(d) (3 × 1) × 5 = 3 × 5 = 17
(e) (7 × 0) × 2 = 0 × 2 = 0
(f) 7 × (0 × 2) = 7 × 0 = 0
(g) 6 × (1 × 5) = 6 × 5 = 36
(h) (6 × 1) × 5 = 6 × 5 = 36

16. (a) and (c).

17. (a) $$\begin{aligned}(A \cup B) \cup C &= \{a, b, c, d, e, f\}\\ &\quad \cup \{a, b, c, g, h, k\}\\ &= \{a, b, c, d, e, f, g, h, k\}\end{aligned}$$

(b) $$\begin{aligned}A \cup (B \cup C) &= \{a, b, c, d\}\\ &\quad \cup \{a, b, c, e, f, g, h, k\}\\ &= \{a, b, c, d, e, f, g, h, k\}\end{aligned}$$

$(A \cup B) \cup C = A \cup (B \cup C)$

(c) $$\begin{aligned}(A \cap B) \cap C &= \{a, b, c\}\\ &\quad \cap \{a, b, c, g, h, k\}\\ &= \{a, b, c\}\end{aligned}$$

(d) $$\begin{aligned}A \cap (B \cap C) &= \{a, b, c, d\} \cap \{a, b, c\}\\ &= \{a, b, c\}\end{aligned}$$

$(A \cap B) \cap C = A \cap (B \cap C)$

18. (a) $$\begin{aligned}(P \cup Q) \cup R &= \{2, 4, 6, 8, 10, 3, 9, 12\}\\ &\quad \cup \{6, 12, 18\}\\ &= \{2, 4, 6, 8, 10, 3, 9, 12, 18\}\end{aligned}$$

(b) $$\begin{aligned}P \cup (Q \cup R) &= \{2, 4, 6, 8, 10\}\\ &\quad \cup \{3, 6, 9, 12, 18\}\\ &= \{2, 4, 6, 8, 10, 3, 9, 12, 18\}\end{aligned}$$

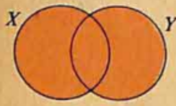
$(P \cup Q) \cup R = P \cup (Q \cup R)$

(c) $$\begin{aligned}(P \cap Q) \cap R &= \{6\} \cap \{6, 12, 18\}\\ &= \{6\}\end{aligned}$$

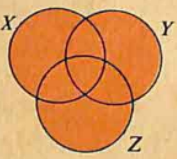
(d) $$\begin{aligned}P \cap (Q \cap R) &= \{2, 4, 6, 8, 10\} \cap \{6, 12\}\\ &= \{6\}\end{aligned}$$

$(P \cap Q) \cap R = P \cap (Q \cap R)$

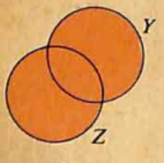
19. (a) Yes.



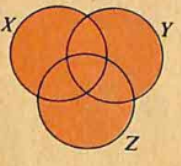
$X \cup Y$



$(X \cup Y) \cup Z$

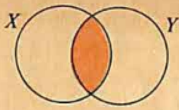


$Y \cup Z$

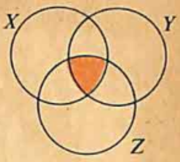


$X \cup (Y \cup Z)$

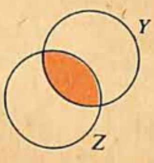
(b) Yes.



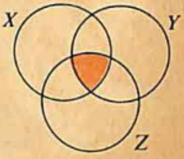
$X \cap Y$



$(X \cap Y) \cap Z$



$Y \cap Z$



$X \cap (Y \cap Z)$

20. (a) $$\begin{aligned}2 \times (5 + 3) &= 2 \times 8\\ &= 16\\ (2 \times 5) + (2 \times 3) &= 10 + 6\\ &= 16\end{aligned}$$

(b) $$\begin{aligned}5 \times (6 + 2) &= 5 \times 8\\ &= 40\\ (5 \times 6) + (5 \times 2) &= 30 + 10\\ &= 40\end{aligned}$$

(c) $$\begin{aligned}4 \times (3 + 0) &= 4 \times 3\\ &= 12\\ (4 \times 3) + (4 \times 0) &= 12 + 0\\ &= 12\end{aligned}$$

(d) $$\begin{aligned}6 \times (5 + 1) &= 6 \times 6\\ &= 36\\ (6 \times 5) + (6 \times 1) &= 30 + 6\\ &= 36\end{aligned}$$

21. (a) $$\begin{aligned}2 \times (1 + 3) &= 2 \times 4\\ &= 13\\ (2 \times 1) + (2 \times 3) &= 2 + 11\\ &= 13\end{aligned}$$

(b) $$\begin{aligned}4 \times (2 + 1) &= 4 \times 3\\ &= 22\\ (4 \times 2) + (4 \times 1) &= 13 + 4\\ &= 22\end{aligned}$$

(c) $$\begin{aligned}3 \times (0 + 2) &= 3 \times 2\\ &= 11\\ (3 \times 0) + (3 \times 2) &= 0 + 11\\ &= 11\end{aligned}$$

(d) $$\begin{aligned}2 \times (3 + 2) &= 2 \times 10\\ &= 20\\ (2 \times 3) + (2 \times 2) &= 11 + 4\\ &= 20\end{aligned}$$

ANSWERS

22. (a) $2 \times (4 - 1) = 2 \times 3$
$= 6$
$(2 \times 4) - (2 \times 1) = 8 - 2$
$= 6$

(b) $3 \times (3 - 2) = 3 \times 1$
$= 3$
$(3 \times 3) - (3 \times 2) = 9 - 6$
$= 3$

(c) $2 \times (5 - 2) = 2 \times 3$
$= 6$
$(2 \times 5) - (2 \times 2) = 10 - 4$
$= 6$

(d) $3 \times (6 - 3) = 3 \times 3$
$= 9$
$(3 \times 6) - (3 \times 3) = 18 - 9$
$= 9$

Multiplication is distributive over subtraction for the natural numbers.

23. For the natural numbers:
(a) Subtraction is not distributive over multiplication.
(b) Division is not distributive over addition.
(c) Addition is not distributive over multiplication.

24. (a) (i) $7 + 0 = 7$
(ii) $0 + 5 = 5$
(iii) $18 + 0 = 18$

(b) (i) $0 + 4 = 4$
(ii) $3 + 0 = 3$
(iii) $0 + 10 = 10$

(c) (i) $2 + 0 = 2$
(ii) $0 + 1 = 1$
(iii) $12 + 0 = 12$

25. (a) (i) $7 \times 1 = 7$
(ii) $1 \times 5 = 5$
(iii) $18 \times 1 = 18$

(b) (i) $1 \times 4 = 4$
(ii) $3 \times 1 = 3$
(iii) $1 \times 10 = 10$

(c) (i) $2 \times 1 = 2$
(ii) $1 \times 1 = 1$
(iii) $12 \times 1 = 12$

26. (a) (i) $7 \times 0 = 0$
(ii) $0 \times 5 = 0$
(iii) $0 \div 4 = 0$
(iv) $3 \div 0$ has no answer.

(b) (i) $0 \times 4 = 0$
(ii) $3 \times 0 = 0$
(iii) $0 \div 2 = 0$
(iv) $1 \div 0$ has no answer.

S.C.E.R.T., West Bengal
Library
Calcutta
25/3, B. C. Road

Form No. 3.

PSY, RES.L-1

Bureau of Educational & Psychological Research Library.

The book is to be returned within the date stamped last.

WBGP-59/60-5119C-5M

Form No. 4

BOOK CARD

Coll. No.............. Accn. No......

Author..............................

Title...............................

Date.	Issued to	Returned on

517.521
PAL

THE NEW MATHEMATICS FOR PRIMARY TEACHERS

207

A series of short books giving teachers and student-teachers a new mathematical background so that they can introduce modern mathematics in the primary school. The first five books provide a background to the first three years of mathematical work in the primary school, while the later titles cater for the whole primary range.

The first six titles published are:

1. **Numbers and Number Systems**
2. **Operations with Sets and Numbers**
3. **Finding out about Shapes**
4. **Measures and Measuring**
5. **Working with Numbers**
6. **Relations and Graphs**

ISBN 0 19 919012 7

459